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AN INVENTORY MODEL FOR DETERIORATING PRODUCTS WITH LEAD TIME AND SELLING PRICE SENSITIVE DEMAND UNDER TWO LEVEL STORAGE FACILITY

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Abstract - **In the classical economic ordered quantity model, researchers have developed the inventory models assuming that demand and deterioration are constant. However in practical situation it is not true. In this study we have investigated inventory system for a single deteriorating product under two storage systems where we assumed that the rate of demand is depending on sales price when the quantity of ordered items becomes greater than capacity to store in own facility then two warehouses are included in this system. Any surplus inventory is kept in a rented warehouse which charges higher holding cost than own warehouse. First, the company's own warehouse is fully utilized then the rest is kept in rented warehouse. Thus the goods in rented warehouse initially decrease to zero mainly because of demand and partly due to deterioration. At that time, a bit of stock in own warehouse decreases because of deterioration only. Then the stock in own warehouse decreases to zero due to demand and deterioration. In this study, shortages are acceptable during the lead time and the same is fully backlogged. Numerical examples have been performed to clarify the applicability of this study. Finally, sensitivity analysis has been presented to study the impact of different parameters.**

Keyword - **EOQ, Lead time, two warehouse system, deterioration, price dependent demand, shortages.**

I. INTRODUCTION:

Inventory model deal with the time, at which productions/orders for certain products are to be placed, and the quantity of the production/order.One of the main significant concerns of the management is to make a decision how much and when to order or to produce so that the total inventory cost the system must be minimum.

In traditional inventory models, researchers assumed the market demand rate as constant. In reality, market demand for material goods might be depending on time or selling-price or stock. In an inventory management system, sales-price of products plays a vital function. Duari and Chakrabarti (1) developed economic ordered quantity model with selling-price related demand under shortages. You, (2) developed an EOQ model with sales-price and time sensitive demand rate. Lin et al. (3) studied an inventory model with time varying holding cost and shortages. Many researchers like, Saha & Chakrabartin (14), Rekha et al. (15) developed economic ordered quantity models by assuming the demand rate depends on the selling price.

In practical scenario, when suppliers offer price discount for mass purchase or the items are seasonal, the retailers might buy more products than they can store in their own storehouse. Then they can take a rented warehouse (RW) to keep the surplus unit of products. Usually, holding cost of the rented warehouse is higher than the holding cost of own warehousing because of extra maintenance cost, material handling cost, etc. To decrease the total inventory cost, it will be cost-effective to consume the items from rented warehouse at first. Accordingly, the firms store products in own warehouse before ranted warehouse, but clear the stock in ranted warehouse before own warehouse. Depending on this process, Malik et al. (4) studied an EOQ model under two level storage facilities, where they assumed that the demand rate is stock dependent. Sing and Malik (5) developed a production inventory model by assuming exponential demand and variable deterioration under two-storehouse facility. Duari and Chakrabarti (6) designed an inventory model for deteriorating items involving two warehouses with power-form stock sensitive demand and shortages. Other remarkable models in this trend are designed by Zhou and Yang (7), Jaggi and Tiwari (8), Lee (9) and Yang (10). Other most current publications in this way are developed by Sett et al. (11), Tiwari et al. (12) and Jaggi et al. (13).

Lead time is a very important topic in inventory management system. Hsiao and Lin [17] investigated a buyer vendor ordered quantity model with changeable lead time. Also, Saha and Chakrabarti [16] developed a buyer-vendor supply chain model by assuming lead time as a decision variable. Nowadays, lead time has become an interesting topic for many authors like Ouyang et al. [18], Chang et al. [19], Sarkar et al. [20]. Though, in several realistic situations lead time can be decreased by adding extra cost. By reducing lead times, one can improve customer service and can achieve reduction of safety stock. The additional cost of decreasing lead time consists mostly due to managerial costs, carrying cost since the product's shipment period from the supplier is a key factor of lead time and the speed-up cost of supplier. In the 2016, Maragatham & Palani (21) extended this work.

In this article, we assume a general economic ordered quantity model for deteriorating products with sales-price sensitive market demand rate under two level storage facilities. At the period of lead time shortages are allowed and which is fully backlogged. An investigative solution of this study is expressed and it is explained through numerical examples.

II. ASSUMPTIONS AND NOTATIONS:

This model is developed under the following notations and assumptions:

Notations:

a. $D(p)$: Demand rate per unit time.
b. w: Storage capacity of Owned Warehouse
c. A: The ordering cost of inventory per order.
d. c: purchasing cost per unit
e. T: Length of each replenishment cycle.
f. $I_R(t)$: Inventory at any time t in rented warehouse.
g. $I_o(t)$: Inventory at any time t in owned warehouse.
h. h_o: per unit holding cost (per unit time) in owned warehouse
i. h_r: per unit holding cost (per unit time) in rented warehouse with $h_r \geq h_o$
j. α: rate of deterioration in owned warehouse, where $0 < \alpha < 1$
k. β: rate of deterioration in rented warehouse, where $0 < \beta < 1$
l. t_w: time at which on hand inventory drops to zero in rented warehouse
m. t_1: time at which on hand inventory drops to zero
n. Q: Maximum inventory level
o. p: selling price per unit
p. s: shortage cost per unit

Assumptions:

a. The planning horizon is finite.
b. The rate of market demand $D(p)$ is price dependent and is expressed as $D(p) = ap^{-b}, a, b > 0$
c. Cost of the product remains invariable irrespective of lot size
d. During lead time, shortages are allowed.
e. Rate of replenishment is infinite and lead time is constant.
f. Single deteriorating items have been considered and the rate of deterioration is constant.
g. There is no replenishment or repair of the deteriorated products throughout the inventory cycle.
h. In each inventory cycle we can replenish inventory only one time.
i. Shortages are allowed for the duration of lead time.
j. The ordering amount is $Q_1 = Q + LD(p)$, when $t = L$.
k. Fixed capacity of the own warehouse is w units and the capacity of rented warehouse is unlimited.
l. Products in own warehouse are depleted only after depleting the products stored in rented warehouse.
m. Inventory cost (counting the cost of holding and the cost of deterioration) in rented warehouse is greater than the same in owned warehouse, i.e., $h_r + \beta c > h_o + \alpha c$.

III. MATHEMATICAL MODEL:

The problem has been discussed at this point is how the retailer knows whether or not to hire a storehouse to keep excessive products. If the ordered amount Q is less or equal to the capacity w of own warehouse, then rented warehouse is not needed. But if the ordered lot $Q$is greater than w, then w unit of products are kept in own-warehouse and the leftovers is stored in rented warehouse..

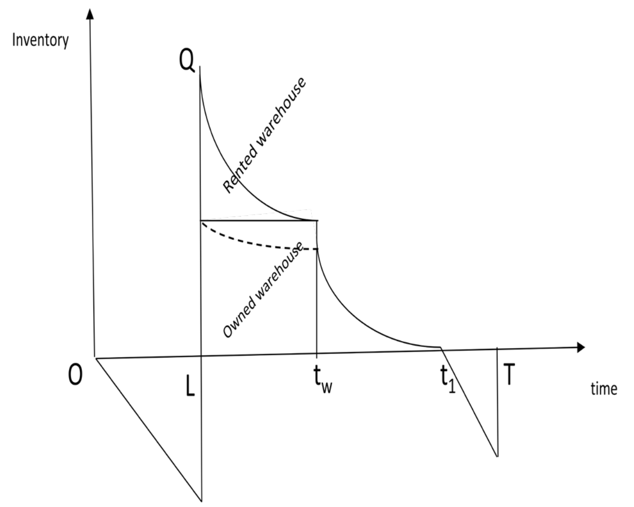

Fig. 1 Inventory-time diagram

Development of Two warehouse Model ($Q > w$):

If $Q$is greater than w, then the inventory system includes two warehouse facility. Initially w unit of products are kept in own warehouse and the excessive amount are stored in rented warehouse. In each cycle, reduction of inventory level occurs because of demand and deterioration. So, during the time interval (L, t_w) products in rented warehouse depleted mainly due to market demand and partly due to deterioration until its level reaches to zero. At that time, a bit of inventory in own warehouse decreases because of deterioration only. Then during (t_w, t_1) inventory in own warehouse depletes because of deterioration and demand. After $t = t_1$shortages start and build up to level S_1, at $t = T$. This behaviour of this system is exposed in Fig. 1. This behaviour can be expressed by the following equations:

$$\frac{dI_R(t)}{dt} + \beta I_R(t) = -ap^{-b}, \quad L \le t \le t_w$$
(1)

$$\frac{dI_o(t)}{dt} = -\alpha I_o(t), \qquad L \le t \le t_w$$
(2)

$$\frac{dI(t)}{dt} + \alpha I(t) = -ap^{-b}, \quad t_w \le t \le t_1$$
(3)

$$\frac{dI(t)}{dt} = -ap^{-b}, \qquad t_1 \le t \le T \qquad (4)$$

With the boundary conditions

$$I_R(t_w) = 0 = I(t_1), I_o(L) = w,$$
$$I(T) = -S_1 \qquad (5)$$

Solving the equations (1)-(4) by using the condition (5) we obtain,

$$I_R(t) = \frac{ap^{-b}}{\beta}\left(e^{\beta(t_w-t)} - 1\right), \qquad L \le t \le t_w$$
(6)

$$I_o(t) = we^{\alpha(L-t)}, \qquad L \le t \le t_w$$
(7)

$$I(t) = \frac{ap^{-b}}{\alpha}\left(e^{\alpha(t_1-t)} - 1\right), \qquad t_w \le t \le t_1$$
(8)

$$I(t) = ap^{-b}(t_1 - t), \qquad t_1 \le t \le T$$
(9)

The maximum inventory (at time $t = L$)

$$I_m = Q = I_R(L) + I_o(L)$$
$$= \frac{ap^{-b}}{\beta}\left(e^{\beta(t_w-L)} - 1\right) + w$$
(10)

Maximum shortage level at the end of a inventory cycle is

$$I_s = S_1 = -I(T) = ap^{-b}(T - t_1)$$
(11)

Now, total inventory cost will include the following costs.

(a) Ordering cost:

$$OC = A$$

(b) Holding cost:

Holding cost in rented warehouse (H_{RW}) is

$$H_{RW} = h_r \int_L^{t_w} I_R(t)\, dt$$
$$= h_r \int_L^{t_w} \frac{ap^{-b}}{\beta}\left(e^{\beta(t_w-t)} - 1\right) dt$$
$$= \frac{h_r ap^{-b}}{\beta}\left[\frac{e^{\beta(t_w-L)-1}}{\beta} + (L - t_w)\right]$$
(12)

Holding cost in owned warehouse (H_{OW}) is

$$H_{OW} = h_o\left[\int_L^{t_w} I_o(t)\, dt + \int_{t_w}^{t_1} I_o(t)\, dt\right]$$
$$= h_o\left[\int_L^{t_w}\left\{we^{\alpha(L-t)}\right\} dt + \int_{t_w}^{t_1}\left\{\frac{ap^{-b}}{\alpha}\left(e^{\alpha(t_1-t)} - 1\right)\right\} dt\right]$$
$$= h_o\left[\frac{w}{\alpha}\left\{1 - e^{\alpha(L-t_w)}\right\} + \frac{ap^{-b}}{\alpha}\left\{\frac{e^{\alpha(t_1-t_w)-1}}{\alpha} + (t_w - t_1)\right\}\right] \qquad (13)$$

Therefore, the total holding cost (C_H) is,

$$C_H = H_{RW} + H_{OW}$$
$$= \frac{h_r ap^{-b}}{\beta}\left[\frac{e^{\beta(t_w-L)-1}}{\beta} + (L - t_w)\right] + h_o\left[\frac{w}{\alpha}\left\{1 - e^{\alpha(L-t_w)}\right\} + \frac{ap^{-b}}{\alpha}\left\{\frac{e^{\alpha(t_1-t_w)-1}}{\alpha} + (t_w - t_1)\right\}\right]$$
(14)

(c) Purchase cost (C_p):

$$C_p = c[Q + LD(p)]$$
$$= c\left[\frac{ap^{-b}}{\beta}\left(e^{\beta(t_w-L)} - 1\right) + w + Lap^{-b}\right]$$
(15)

(d) Deterioration cost (C_D):

$$C_D = c[Q - D(p)(t_1 - L)]$$
$$= c\left[\frac{ap^{-b}}{\beta}\left(e^{\beta(t_w-L)} - 1\right) + w - ap^{-b}(t_1 - L)\right]$$
(16)

(e) Shortage cost (C_s):

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$$C_s = s\left[-\int_{t_1}^{T} I(t)dt\right]$$
$$= -s\left[\int_{t_1}^{T} ap^{-b}(t_1 - t)\, dt\right]$$
$$= \frac{sap^{-b}}{2}(T - t_1)^2 \quad (17)$$

Therefore, the total inventory cost (TIC):

$$TIC = OC + C_H + C_p + C_D + C_s$$
$$= A + \frac{h_r ap^{-b}}{\beta}\left[\frac{e^{\beta(t_w - L)} - 1}{\beta} + (L - t_w)\right] + h_o\left[\frac{w}{\alpha}\left\{1 - e^{\alpha(L - t_w)}\right\} + \frac{ap^{-b}}{\alpha}\left\{\frac{e^{\alpha(t_1 - t_w)} - 1}{\alpha} + (t_w - t_1)\right\}\right] + c\left[\frac{ap^{-b}}{\beta}\left(e^{\beta(t_w - L)} - 1\right) + w + Lap^{-b}\right] + c\left[\frac{ap^{-b}}{\beta}\left(e^{\beta(t_w - L)} - 1\right) + w - ap^{-b}(t_1 - L)\right] + \frac{sap^{-b}}{2}(T - t_1)^2 \quad (18)$$

Our aim is to conclude the optimal value of Q to minimize the total cost (TIC). Solving, $\frac{\partial TIC}{\partial t_1} = 0$ we get the value of t_1 which satisfying the condition

$$\left(\frac{\partial^2 TIC}{\partial {t_1}^2}\right) > 0$$

We use LINGO 17.0 software to find the optimum result of the total cost (TIC).

IV. NUMERICAL EXAMPLES:

We consider the following numerical values of the parameters with appropriate units to analyze the article:

$A = 300, a = 2, b = 1, c = 9, \alpha = 0.02, \beta = 0.8, L = 5, h_r = 0.5, h_o = 0.2, s = 7, t_w = 20,$
$w = 1000, p = 12, T = 100$ in appropriate units.

We obtained the optimum value of $t_1 = 96.161, Q = 34907.04$ and minimum total cost is $TIC = 652460.2$

V. SENSITIVITY ANALYSIS:

Sensitivity analysis is conducted by changing one of the parameters $h_o, h_r, \alpha, \beta, a, b$ and keeping the left over parameters unaffected.

Table - 1 Impact of h_o on the total cost

Change value		t_1	TIC
h_o	0.3	94.013	653838.9
	0.25	95.059	653150.3
	0.2	96.161	652460.2
	0.15	97.323	651768.4
	0.1	98.562	651074.7

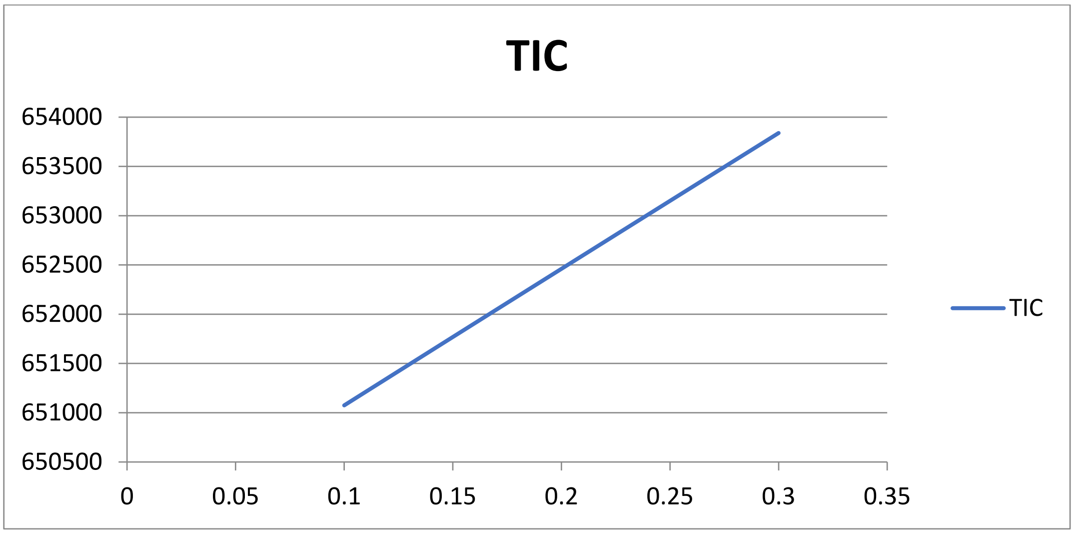

Fig.2. Graph of h_0 Vs TIC

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Table - 2 Impact of h_r on the total cost

Change value		**t_1**	**TIC**
h_r	0.75	96.161	663055.4
	0.625	96.161	657757.8
	0.5	96.161	652460.2
	0.375	96.161	647162.6
	0.25	96.161	641865.0

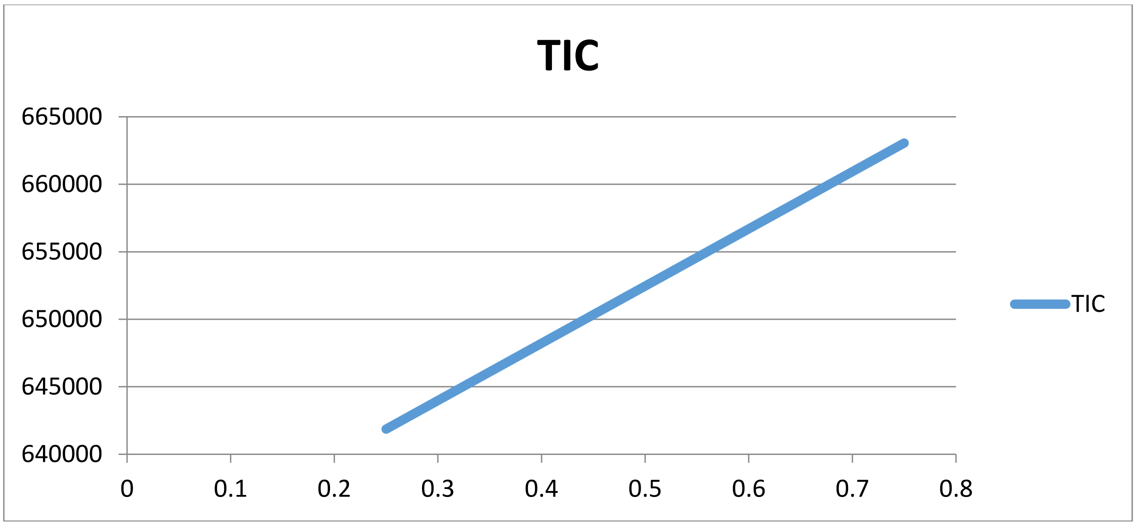

Fig.3. Graph of h_r Vs TIC

Table - 3 Impact of α on the total cost

Change value		**t_1**	**TIC**
α	0.03	93.579	652349.5
	0.025	94.980	652400.3
	0.02	96.161	652460.2
	0.015	97.133	652529.2
	0.01	97.915	652607.0

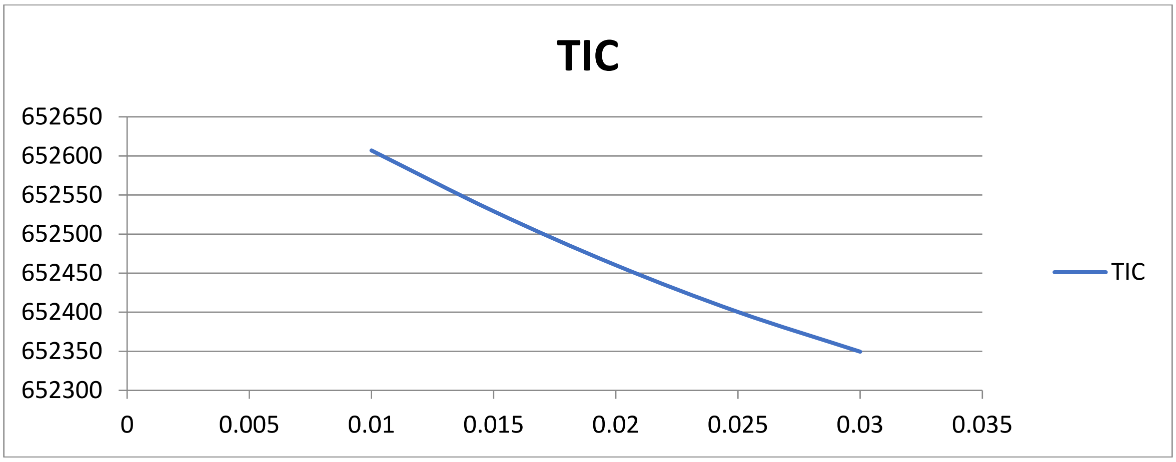

Fig.4. Graph of α Vs TIC

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Table - 4 Impact of β on the total cost

Change value		t_1	TIC
β	0.95	96.161	5039862.0
	0.9	96.161	2527369.0
	0.8	96.161	652460.2
	0.75	96.161	339846.6
	0.7	96.161	182756.9

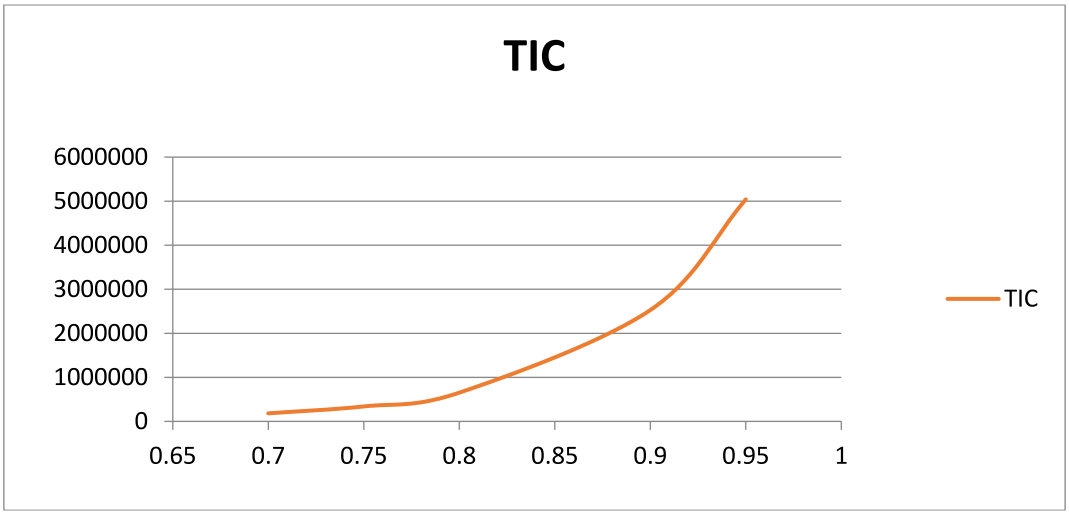

Fig.5. Graph of β Vs TIC

Table - 5 Impact of a on the total cost

Change value		t_1	TIC
a	3	96.161	968244.4
	2.5	96.161	810352.3
	2	96.161	652460.2
	1.5	96.161	494568.1
	1	96.161	336676.0

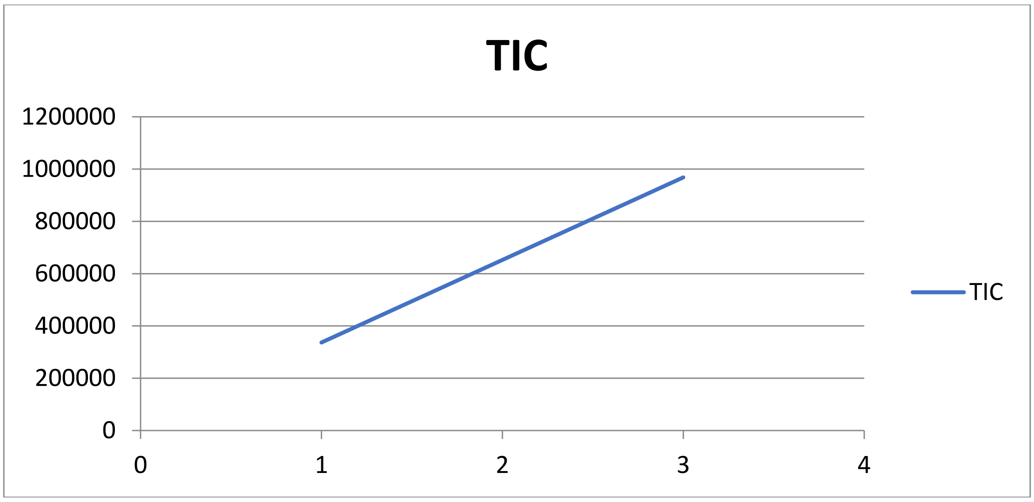

Fig.6. Graph of a Vs TIC

Table - 6 Impact of b on the total cost

Change value		t_1	**TIC**
b	1.5	96.161	203209.9
	1.25	96.161	360224.0
	1	96.161	653460.2
	0.75	96.161	1196373.0
	0.5	96.161	2208709.0

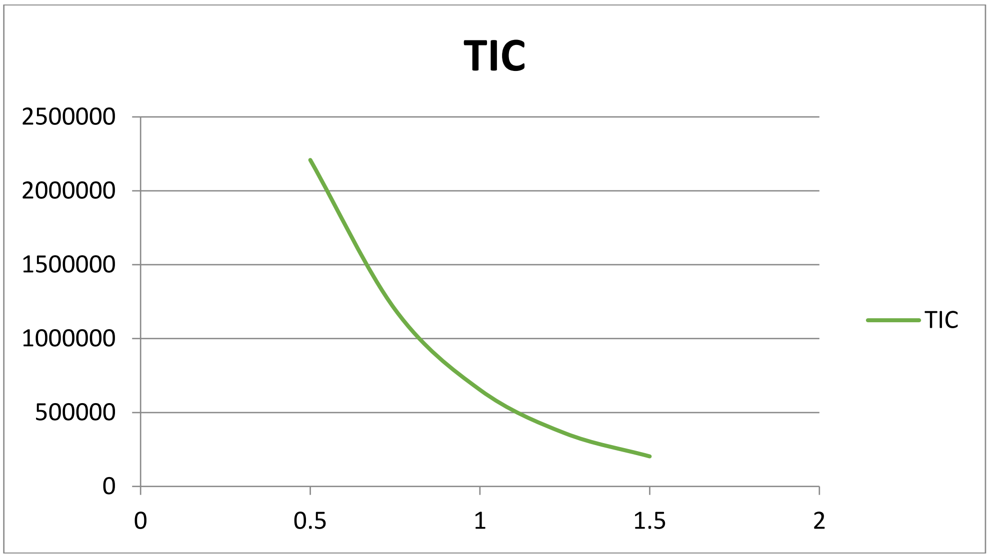

Fig.7. Graph of b Vs TIC

Observations:
We observe from the above tableau that:

i. With the increase (or decrease) of α, b, the total inventory cost TIC decreases (or increases) monotonically. Further, TIC is highly sensitive towards the parameters b but moderately sensitive towards α.

ii. With the decrease (or increase) of h_o, h_r, β, a, the total inventory cost TIC decreases (or increases) monotonically. Further, TIC are highly sensitive to the parameters β, a but moderately sensitive to the parameters h_o, h_r

VI. CONCLUSION

The article designs an inventory model under two level storage facilities for deteriorating products and considers lead time as invariable. The rate of market demand is supposed to be sales-price sensitive. Price is the major factor of the consumer when she/he is going to purchase a product. Shortages are approved during lead time and fully backlogged. Some realistic features like different deterioration rate in own warehouse and ranted warehouse, shortages are fully backlogged are investigated. It has also been supposed that the holding cost in ranted storehouse is higher than in own storehouse. Finally, a numerical example is calculated to illustrate the proposed inventory system. Also, a sensitivity analysis is performed and the results provide important managing information knowledge. In future, researchers can do more work by assuming different types of market demand rate, variable cost etc. Also it can be extended by assuming some parameters as fuzzy.

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